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# Basics of Vehicle Dynamics

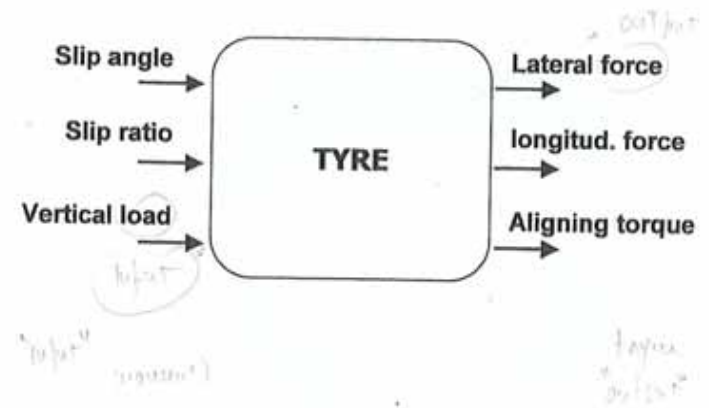
## Module H2

### Tyre Behaviour

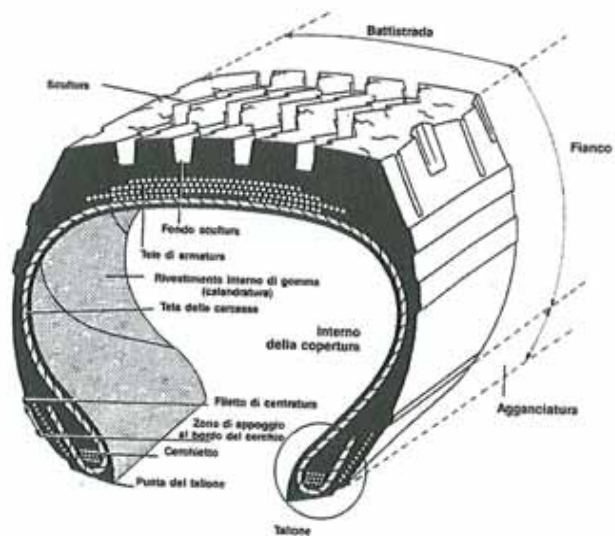
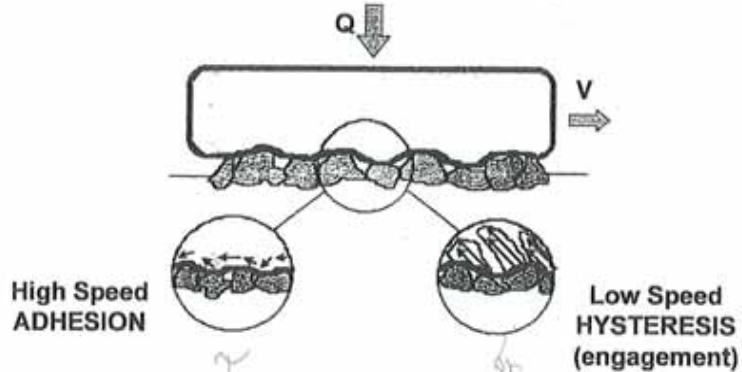
- Forces generated by tyre
  - Tyre effects
  - Tyre transient response
  - Handling tyre models
- 

### TYRES FUNCTIONS ARE:

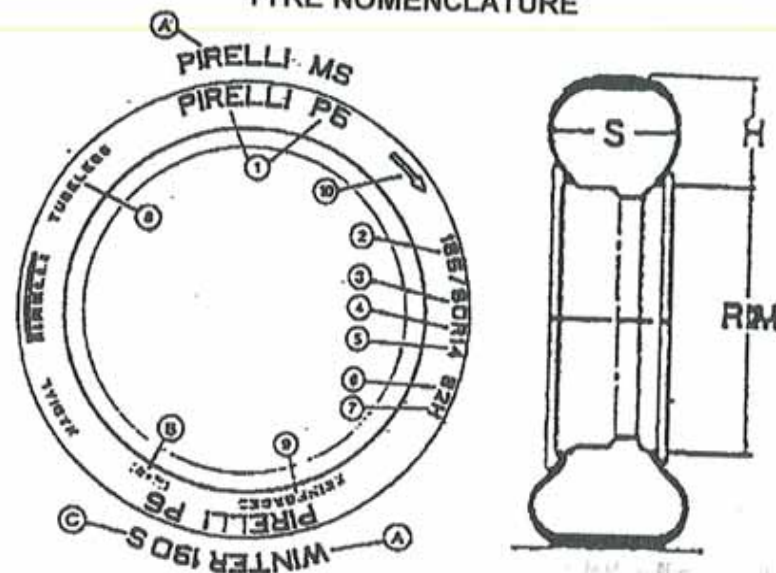
- ❖ Supplying forces for controlling and stabilizing the vehicle *lateral*
- ❖ Transmitting torque to the ground (TRACTION) *longitudinal*
- ❖ Filtering road roughness (COMFORT) *vertical*



ADHESION AND HYSTERESIS



TYRE NOMENCLATURE



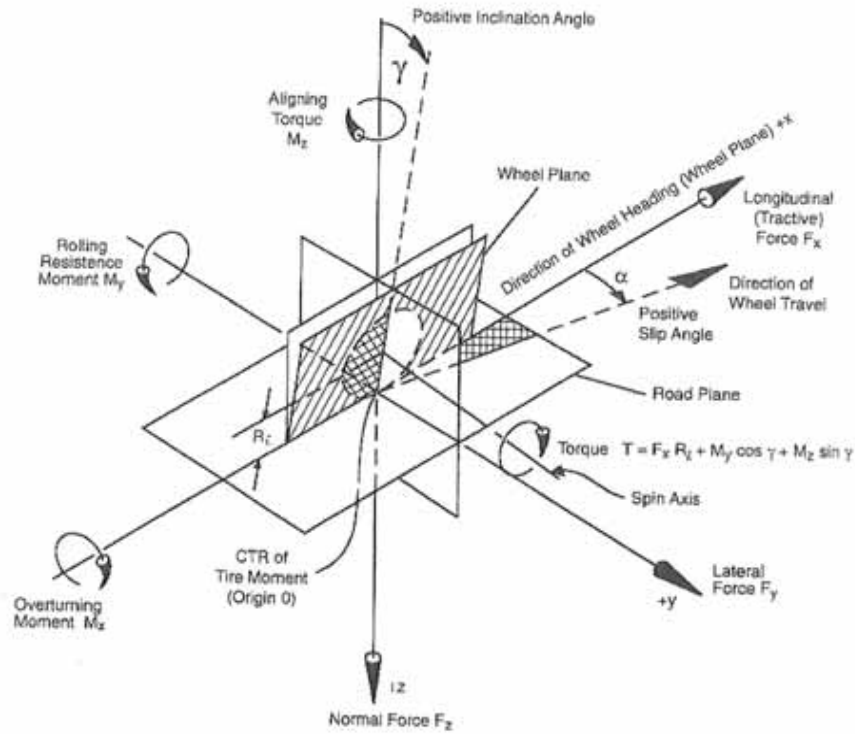
- 1 Manufacturer name and tread type
- 2 Section nominal width in mm
- 3 Ratio between sidewall height  $H$  and section width  $S$ , usually not given if it equals 80
- 4 Type of structure ( R if radial, - conventional)
- 5 Shrinking diameter in inches
- 6 Related to load capacity
- 7 Tyre maximum speed code
- 8 Tubeless tyre. Tyres with tube have the writing TUBE TYPE
- 9 Reinforced internal structure for weight higher than usual
- 10 The arrow shows the preferential rolling direction

Winter tyres marking

- A Manufacturer name and product name
- B "Mud and Snow" indicates a tyre fit to that specific duty
- C Letter S means pits

**TYRE REFERENCE SYSTEM**

**GEOMETRICAL VARIABLES and CONTACT FORCES  
(SAE J670)**



**Forces generated by tyre**

**Lateral Force  
Longitudinal Force  
Aligning Torque**

## LATERAL FORCE

A lateral tire force originates at the "center" of the tire contact with the road, lies in the horizontal road plane and is perpendicular to the direction in which the wheel is headed



A vehicle turns because of the applied lateral tire forces

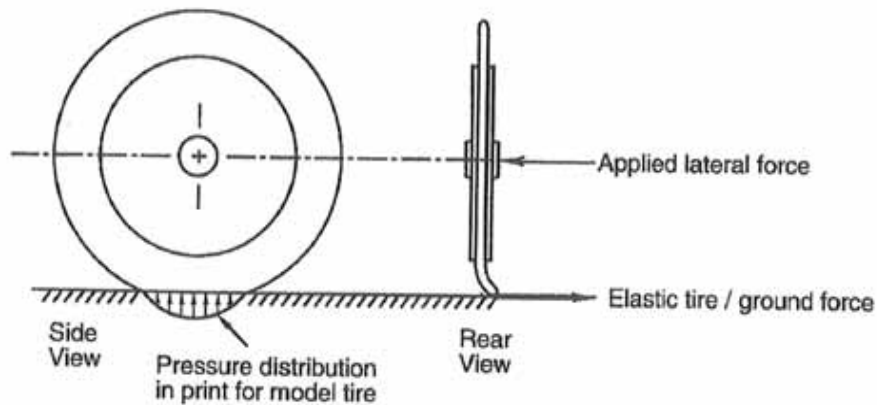


### 1 - How the tyre interacts with the ground

- Constant vertical force
- Lateral force push the tyre



The tyre acts like a spring  
Sidewalls bent elastically



## FORCES AND MOMENTS

The external forces acting on the vehicle can be summed into one force vector having the following components

Longitudinal Force ( $F_x$ ): is the force component in the x-direction

Lateral Force ( $F_y$ ): is the force component in the y-direction

Normal Force ( $F_z$ ): is the force component in the z-direction

The external moment acting on the vehicle can be summed into one moment vector having the following components

Rolling Moment ( $M_x$ ): is the moment component tending to rotate the vehicle about the x-axis, positive clockwise when looking in the positive direction of the x-axis

Pitching Moment ( $M_y$ ): is the moment component tending to rotate the vehicle about the y-axis, positive clockwise when looking in the positive direction of the y-axis

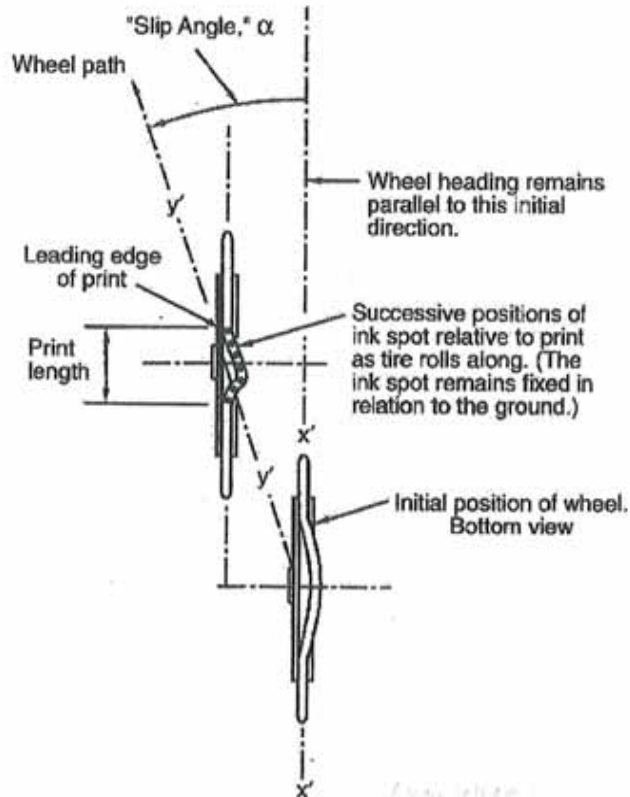
Yawing Moment ( $M_z$ ): is the moment component tending to rotate the vehicle about the z-axis, positive clockwise when looking in the positive direction of the z-axis

2 - How the tyre interacts with the ground

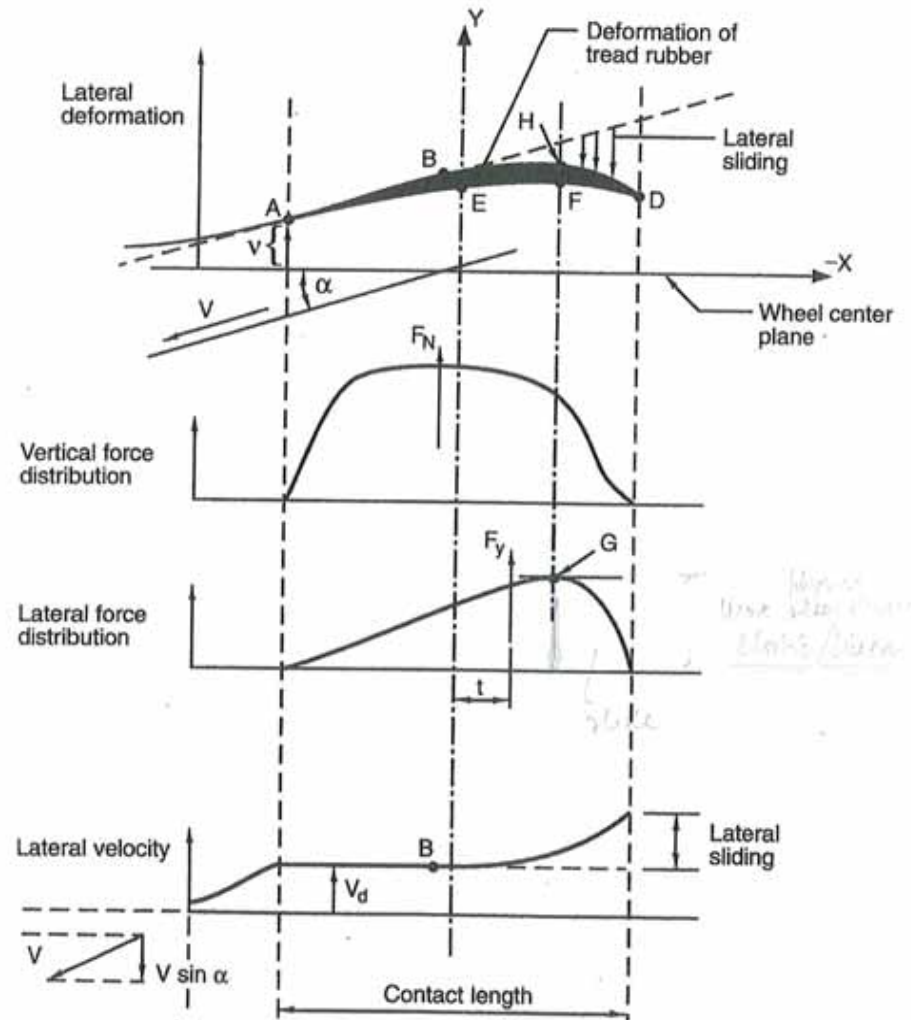
• Tyre is rolled



$\alpha$  angle between velocity vector and the wheel plane  
Each tread unit enters the print and moves aft to the print and also sideways because of the lateral deflection in the print

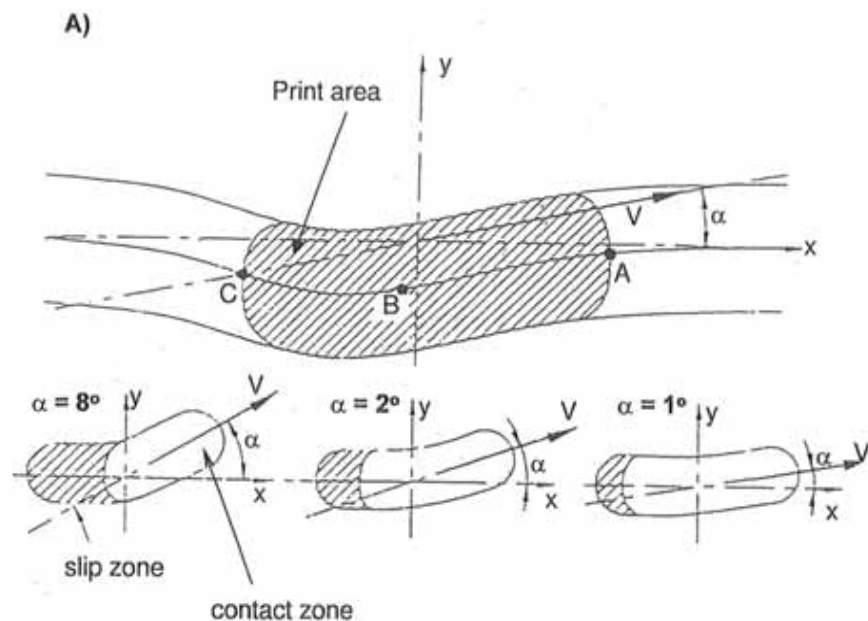


Slip angle increases with lateral force, in a progressive process.  
When the print slides, the lateral force is determined by the coefficient of friction.



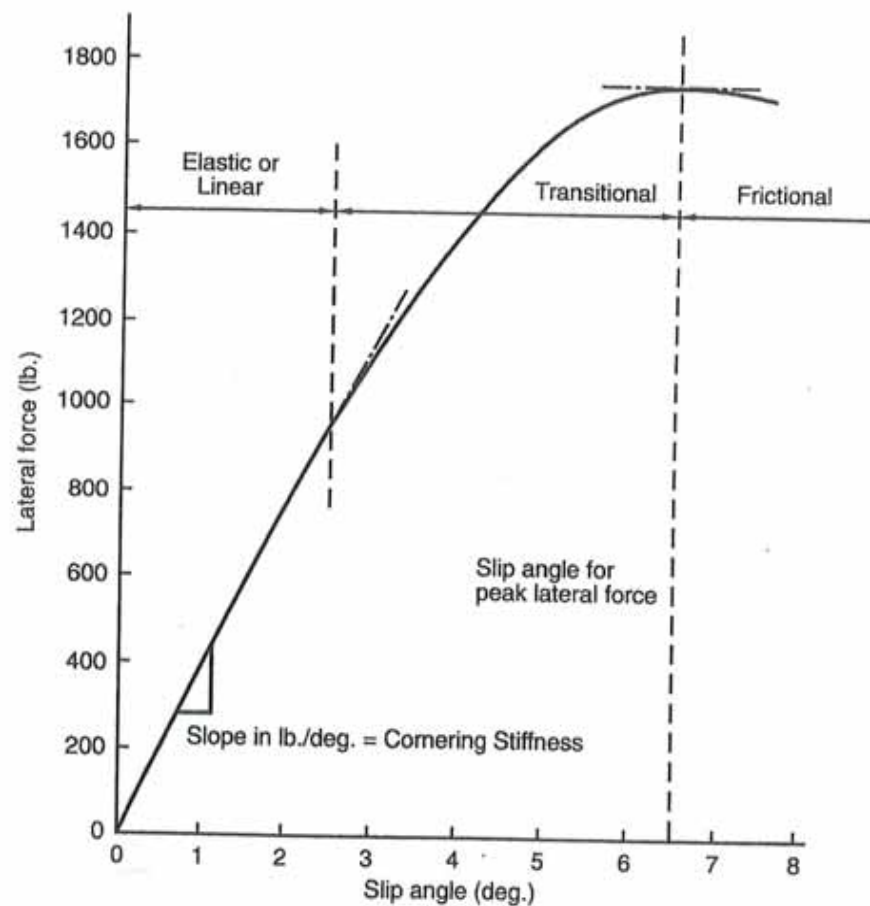
Distribution of forces and lateral velocity over the contact length.

SLIP ANGLE DEFINITION



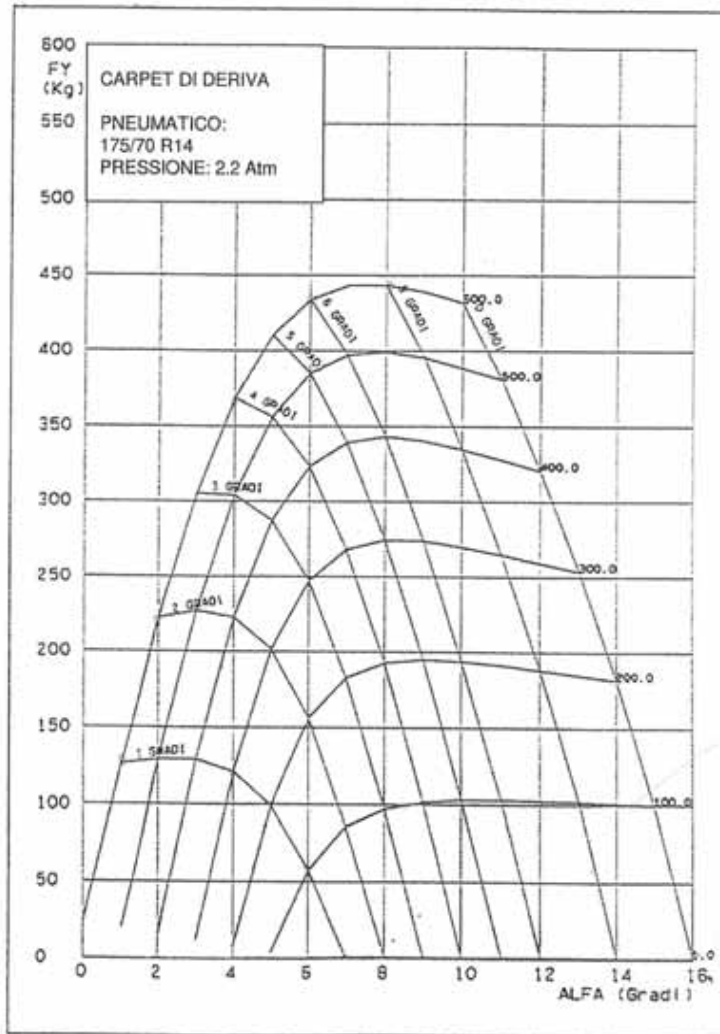
LATERAL FORCE vs SLIP ANGLE

For a given load.



*Lateral force vs. slip angle.*

LATERAL FORCE CARPET



ALIGNING TORQUE

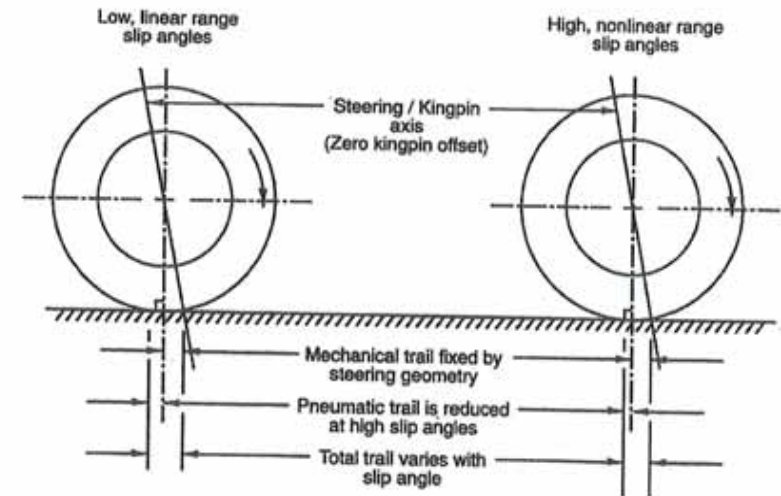
**Aligning Torque** (SAE J670): describe a tyre's tendency to steer about a vertical axis through the "center" of the print. At low and medium slip angles the tyre tends to align its heading with its path.

**Pneumatic Trail:** distance from the fore-aft center of the print to the center of action of the lateral force due to uneven distribution of these. IT CHANGES WITH TYRE OPERATING CONDITIONS.

**ALIGNING TORQUE = LATERAL FORCE TIMES PNEUMATIC TRAIL** ←

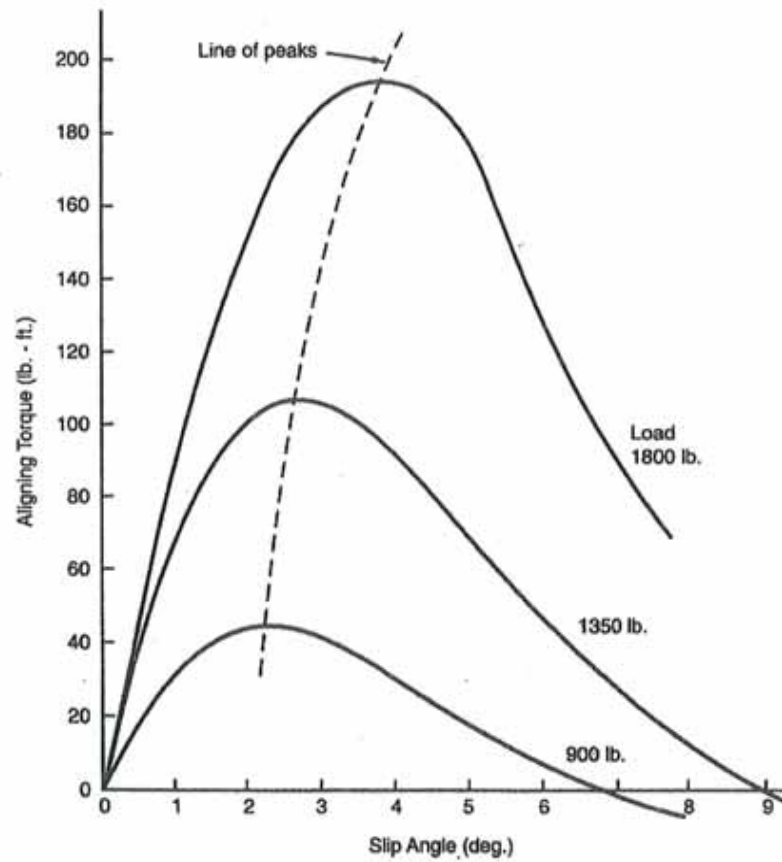
**Mechanical Trail:** steering pivot obtained by caster angle and kingpin offset

**STEERING TORQUE = ( PNEUMATIC\_trail + MECH\_trail)\*FY**



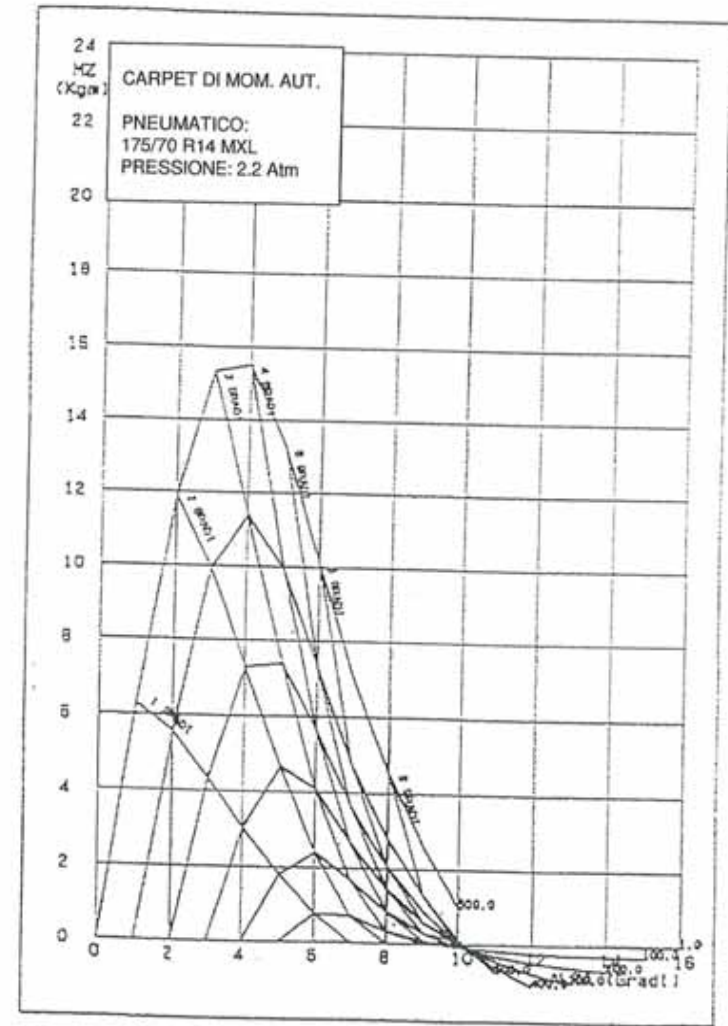
*Pneumatic and mechanical trail*

ALIGNING TORQUE vs SLIP ANGLE

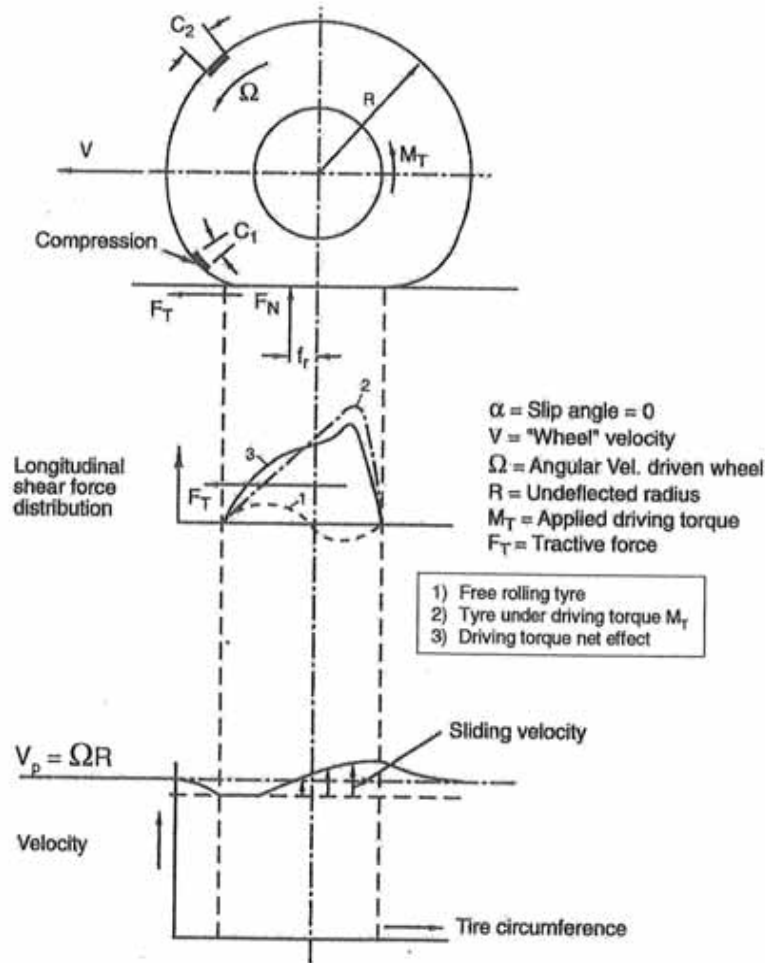


Aligning torque vs. slip angle for several loads.

ALIGNING TORQUE CARPET



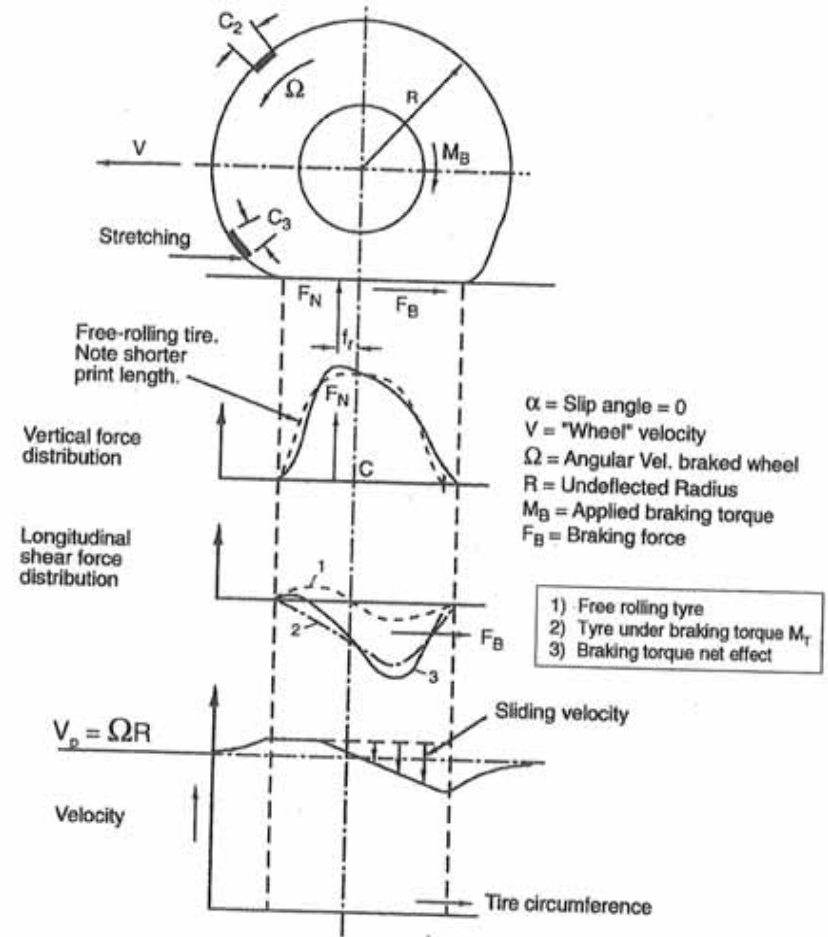
LONGITUDINAL FORCE: TRACTION



Distribution of forces and sliding velocity, over the contact length of a tyre under the action of a driving torque  $M_T$ .

Tyre print characteristics - traction

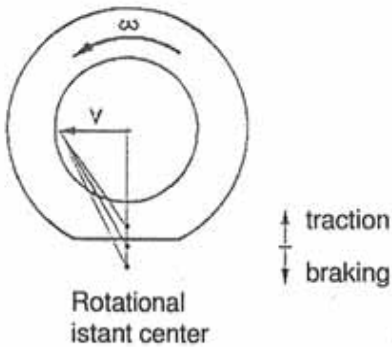
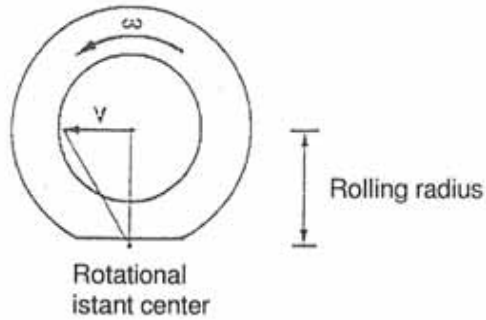
LONGITUDINAL FORCE: BRAKING



Distribution of forces and sliding velocity over the contact length of a tyre under the action of a braking torque  $M_B$ .

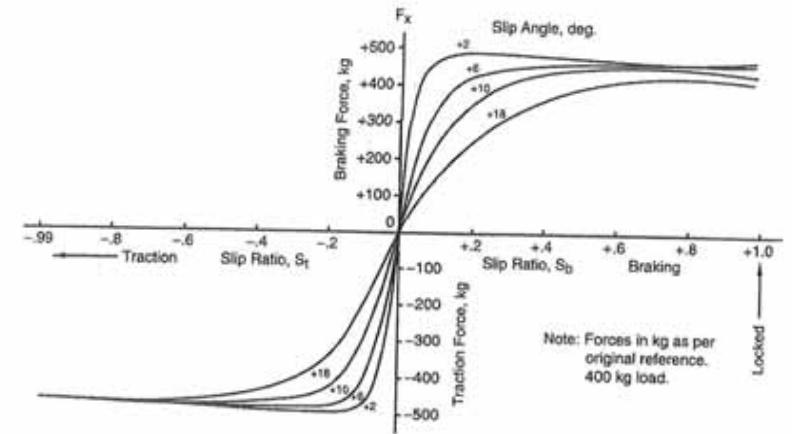
Tyre print characteristics—braking

SLIP RATIO

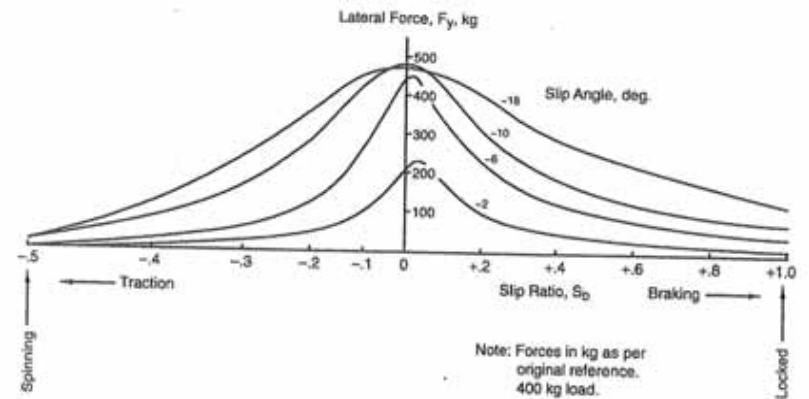


$$S = \frac{\omega \cdot R - V}{\omega \cdot R} \quad \text{traction}$$

$$S = \frac{\omega \cdot R - V}{V} \quad \text{braking}$$

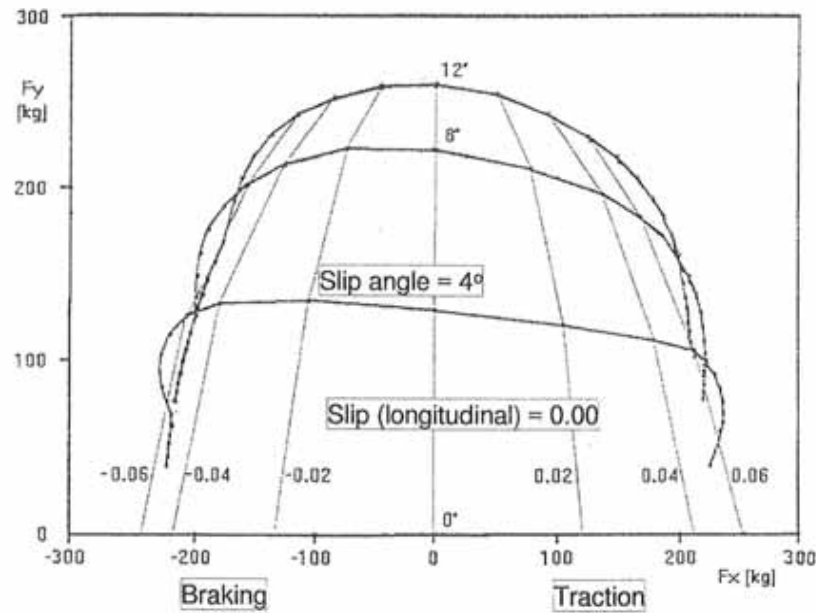


Braking and traction forces vs. slip ratio and slip angle



Lateral force vs. slip ratio and slip angle

### LONGITUDINAL and LATERAL FORCES INTERACTION



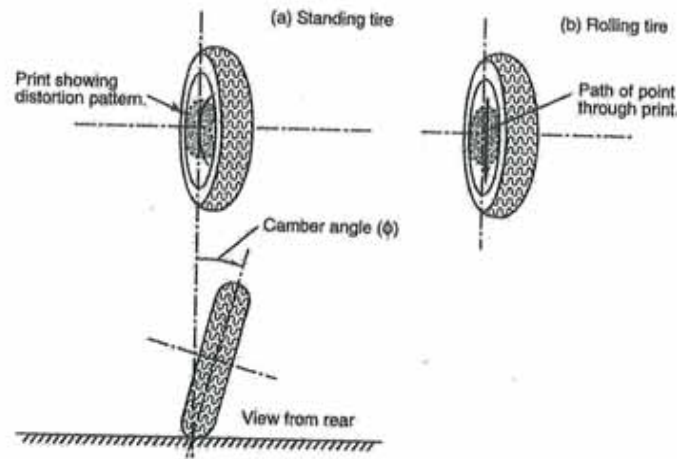
### Tyre Effects

### CAMBER EFFECTS

Camber angle is defined as the angle between a tilted wheel plane and the vertical.



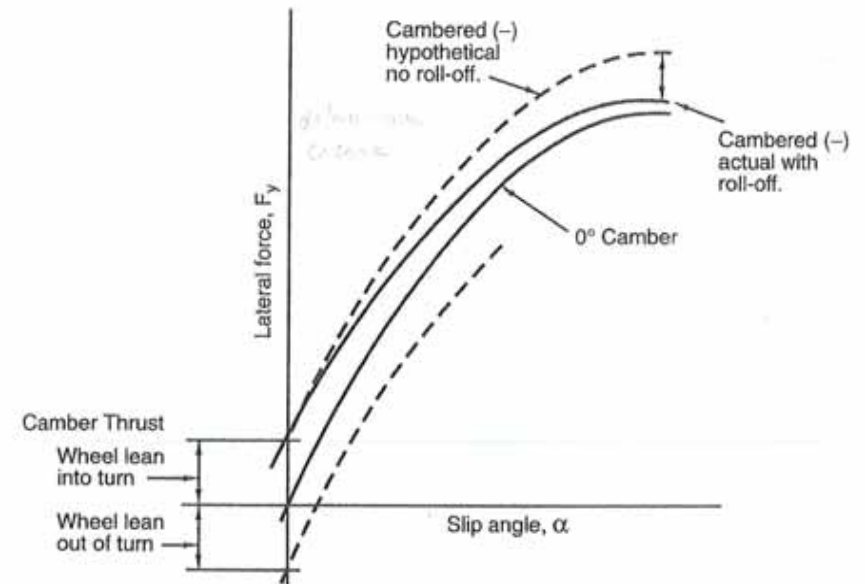
**Camber thrust:** lateral force produced by a cambered rolling pneumatic-tired wheel; it arises from lateral distortion in the print.



*Distortion in print of a tire at a camber angle.*

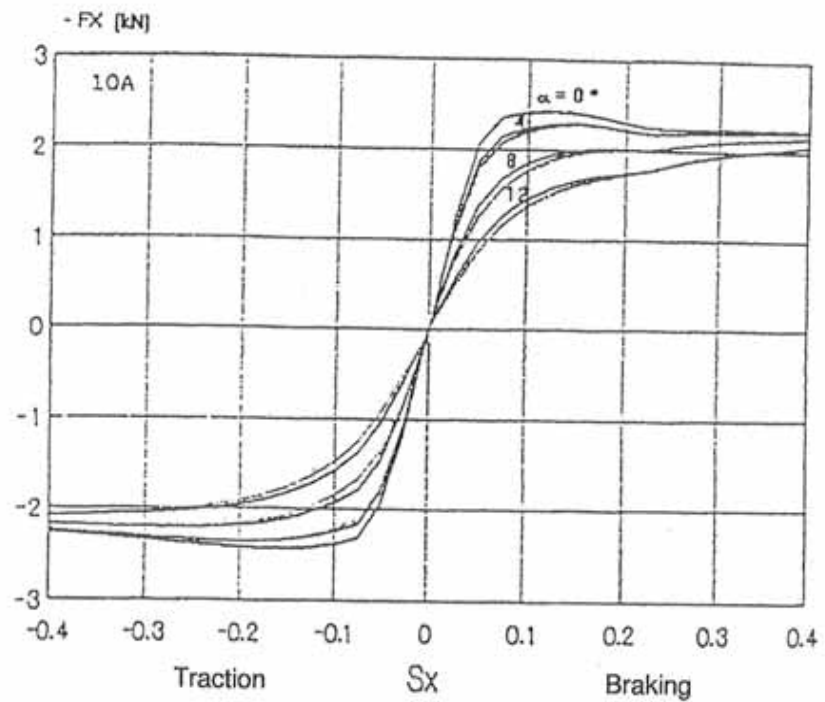
For traditional bias tires cornering stiffness is generally five to six times greater than the camber stiffness.  
For radial tires the effect may be quite small since the lateral stiffness of the belt and the flexibility of radial cord sidewall inhibit the print distortion

### CAMBER EFFECTS

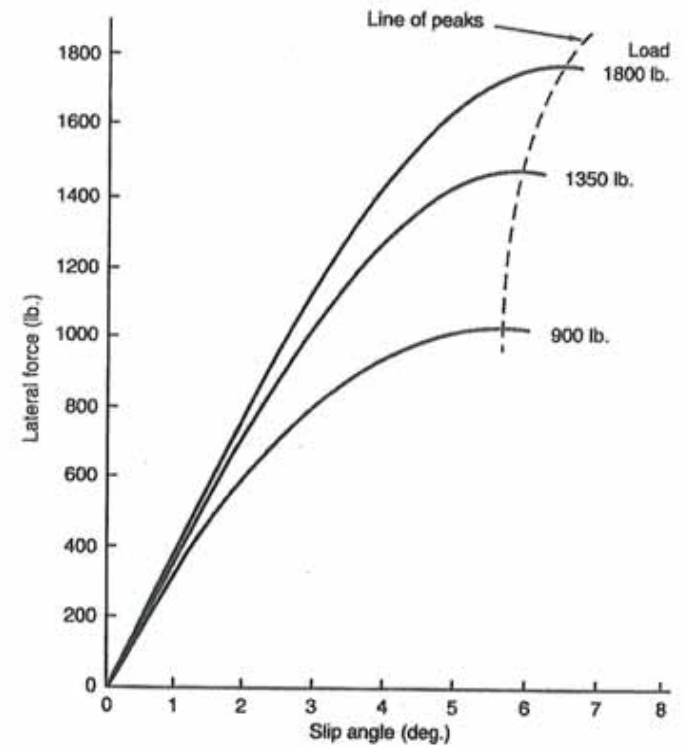


*Camber thrust and camber roll-off at constant load.*

**CAMBER vs.  
LONGITUDINAL FORCE**

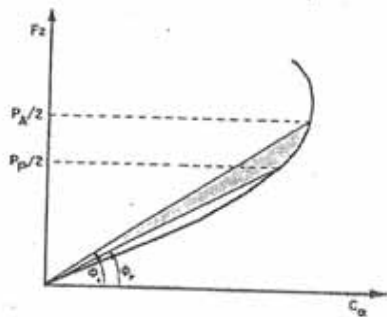
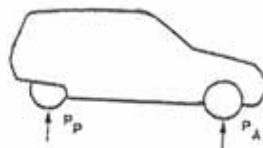
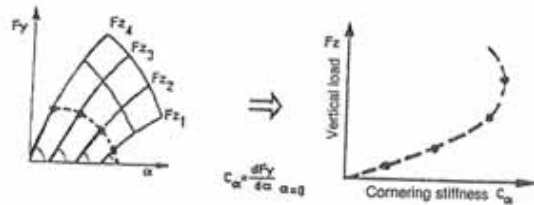


**LOAD EFFECTS  
(Lateral Force)**



*Lateral force vs. slip angle for several loads.*

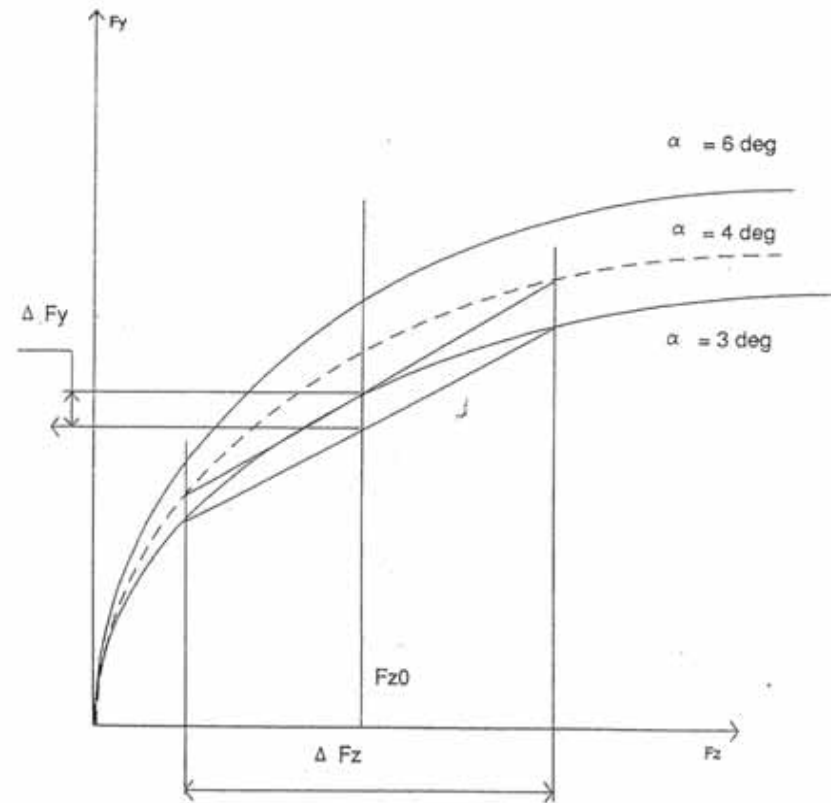
**TYRES UNDERSTEER GRADIENT**



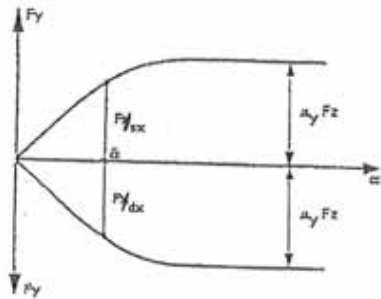
$$KUS = \left( \frac{d\delta_{vol}}{da_y} \right)_{a_y=0} = \left( \frac{d\alpha_A}{da_y} - \frac{d\alpha_P}{da_y} \right)_{a_y=0} \cdot \tau_{vol} =$$

$$= \left( \frac{d(F_{yA}/C_{\alpha A})}{d(2 \cdot F_{yA}/P_A)} - \frac{d(F_{yP}/C_{\alpha P})}{d(2 \cdot F_{yP}/P_P)} \right)_{a_y=0} \cdot \tau_{vol} = \frac{1}{2} \cdot \left( \frac{P_A}{C_{\alpha A}} - \frac{P_P}{C_{\alpha P}} \right) \cdot \tau_{vol} = (tg\phi_A - tg\phi_P) \cdot \tau_{vol}$$

**TYRES NON LINEARITIES vs. VERTICAL LOAD :  
Single axle**

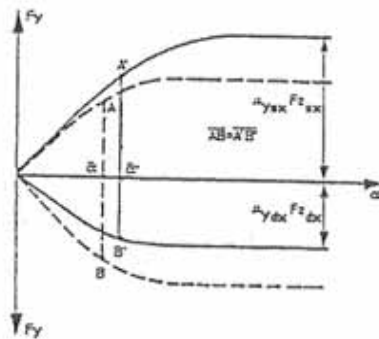


LOAD TRANSFER



NO LOAD TRANSFER

- $\bar{\alpha}$  slip angle to generate load
- $F_y = F_{y_{sx}} + F_{y_{dx}}$
- $F_z$  vertical load on single wheels

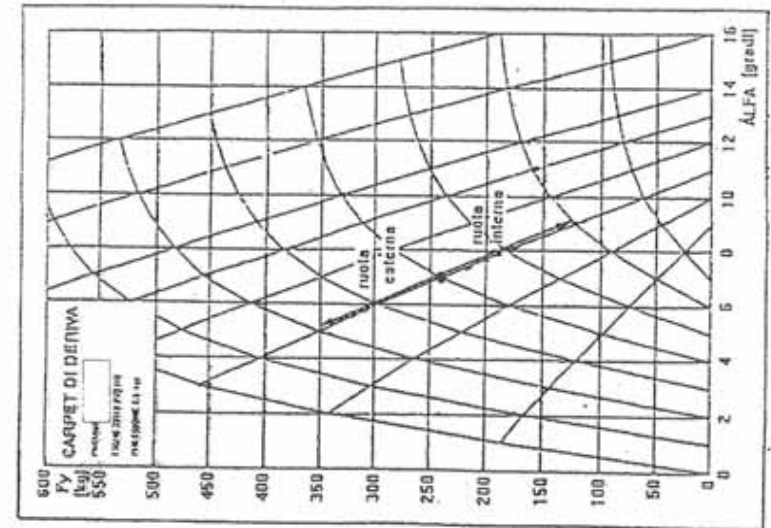
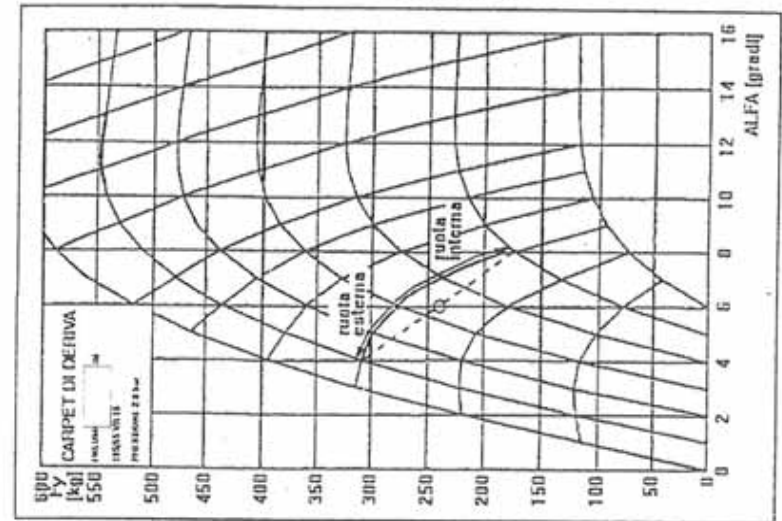


LOAD TRANSFER

- $\bar{\alpha}'$  slip angle to generate load
- $F_y = F_{y_{sx}} + F_{y_{dx}}$
- $F_{z_{sx}}$  vertical load on left wheel
- $F_{z_{dx}}$  vertical load on right wheel

$\bar{\alpha}' > \bar{\alpha}$

TYRE SENSITIVITY vs.  
LOAD TRANSFER



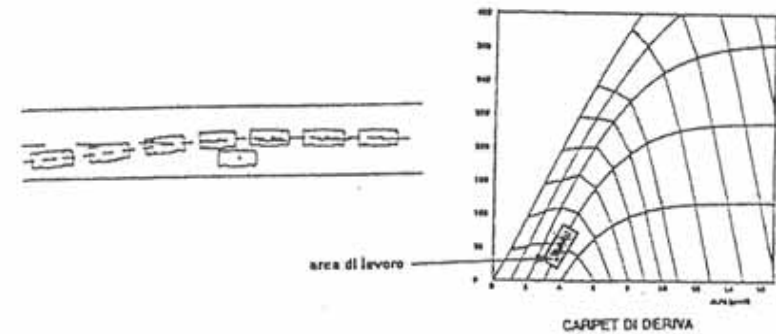
## Tyre transient response

## TYRES DYNAMIC BEHAVIOUR

### SLIP ANGLE TRANSIENT CONDITIONS

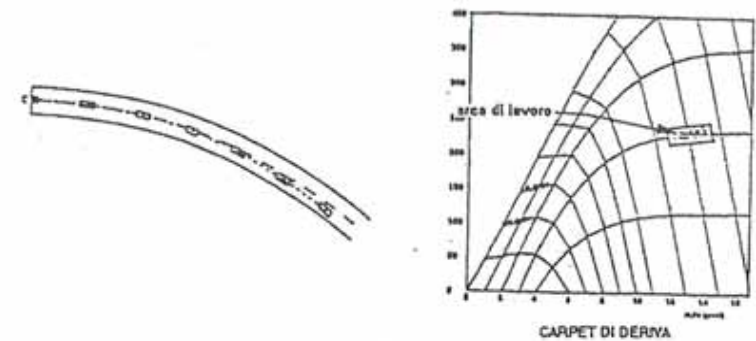
#### NORMAL DRIVING

1. Variable slip angle around small initial values, at roughly constant vertical load



#### LIMIT CONDITION

2. Variable slip angle around large initial values, at roughly constant vertical load

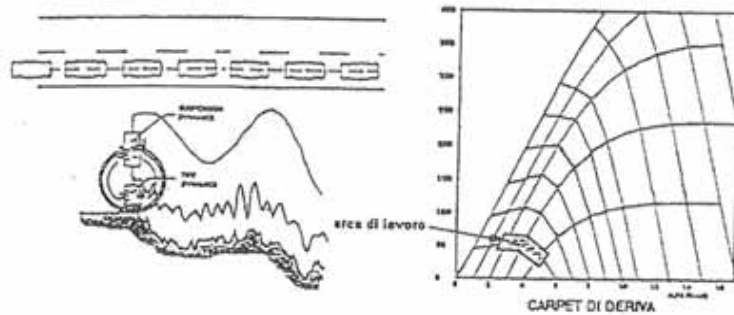


**TYRES DYNAMIC BEHAVIOUR**

**VERTICAL LOAD TRANSIENT CONDITIONS**

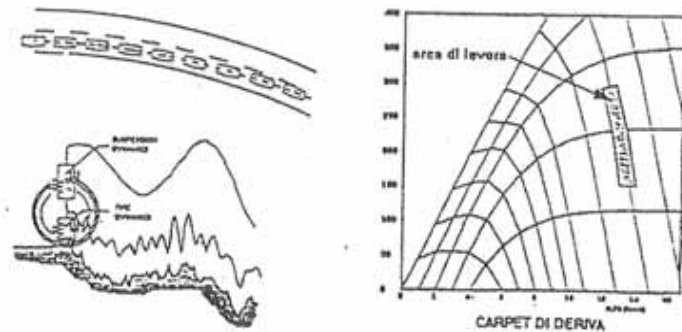
**CENTER FEEL ON UNEVEN ROAD**

1. Variable vertical load, roughly constant small, slip angle



**GRIP ON UNEVEN ROAD**

2. Variable vertical load, roughly constant large slip angle



**TYRE TRANSIENT EQUATION**

$$\dot{F}_y = \frac{V}{S_o(F_z)} \cdot [F_{yss}(\alpha, F_z) - F_y]$$

- $F_y$  = lateral force
- $F_{yss}$  = steady lateral force
- $V$  = speed
- $S_o$  = relaxation length
- $\alpha$  = slip angle
- $F_z$  = vertical load

## Handling Tyre Models

### MODEL GOAL :

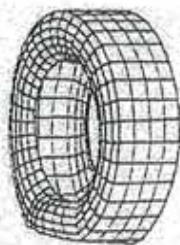
#### *MATHEMATICAL SIMULATION of TYRE BEHAVIOUR in HANDLING*

- Steady state conditions
- Transient conditions
- Different working conditions:
  - High adhesion
  - Low adhesion
  - Pressure
  - Temperature

**MODEL KINDS**

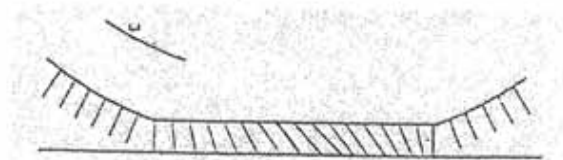
**1) PHYSIC**

To get handling characteristics vs. tyre physic characteristics (dimensions, tread types, carcass structure, ecc.)



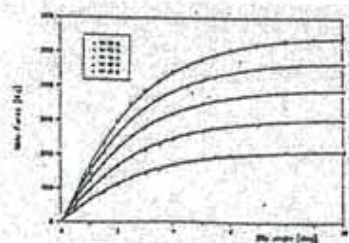
**2) SEMI-PHYSIC**

To get handling characteristics vs. physic equivalent parameters (carcass and tread stiffness, adhesion coeff., ecc.)

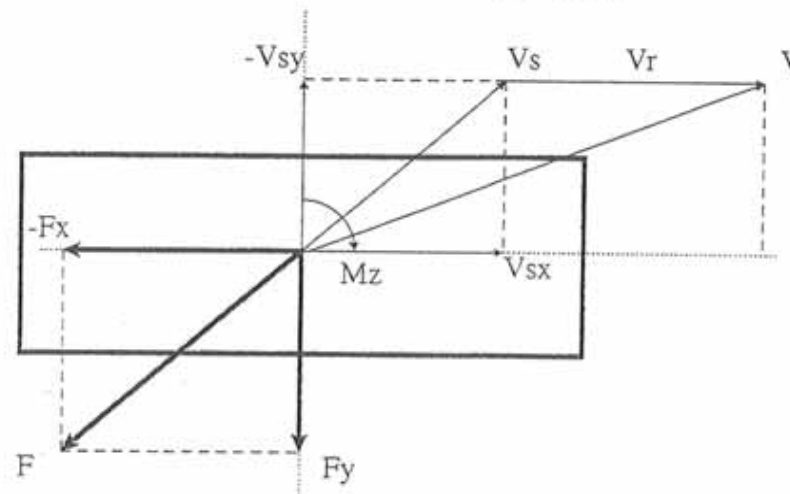


**3) INTERPOLATION**

To get handling characteristics by mathematical interpolation of characteristics obtained by testing or physical models



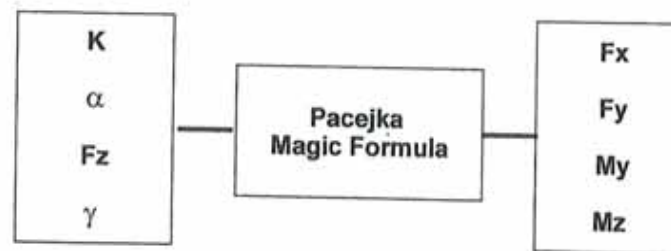
**PACEJKA MAGIC FORMULA**



Tyre view during simultaneous action of lateral and longitudinal loads, showing force and speed vectors

Longitudinal Slip :  $k = \frac{V_{sx}}{V_x}$

Slip angle:  $\tan \alpha = -\frac{V_{sy}}{V_x}$



## VEHICLE APPLIED FORCES

$$Y = D \cdot \sin[C \cdot \arctan(B \cdot (x + Sh)) - E \cdot (B \cdot (x + Sh) - \arctan(B \cdot (x + Sh)))] + Sv$$

dove per  $Y = F_y$  e  $x = \alpha$

$$D = a1 \cdot F_z^2 + a2 \cdot F_z$$

$$B \cdot C \cdot D = a3 \cdot \sin[2 \cdot \arctan(\frac{F_z}{a4})] \cdot (1 - a5 \cdot |\gamma|)$$

$$C = a0 ; \quad E = a6 \cdot F_z + a7$$

$$Sh = a8 \cdot \gamma + a9 \cdot F_z + a10$$

$$Sv = (a112 \cdot F_z^2 + a111 \cdot F_z) \cdot \gamma + a12 \cdot F_z + a13$$

mentre per  $Y = M_z$  e  $x = \alpha$

$$D = c1 \cdot F_z^2 + c2 \cdot F_z$$

$$B \cdot C \cdot D = (c3 \cdot F_z^2 + c4 \cdot F_z) \cdot (1 - c6 \cdot |\gamma|) \cdot e^{-c5 \cdot F_z}$$

$$C = c0 ; \quad E = (c7 \cdot F_z^2 + c8 \cdot F_z + c9) \cdot (1 - c10 \cdot |\gamma|)$$

$$Sh = c11 \cdot \gamma + c12 \cdot F_z + c13$$

$$Sv = (c14 \cdot F_z^2 + c15 \cdot F_z) \cdot \gamma + c16 \cdot F_z + c17$$

## PACEJKA INTERPOLATION MODEL EXAMPLES

