Basics of Vehicle Dynamics

INDEX

Basics of Vehicle Dynamics

Module H1

Introduction to Vehicle Dynamics

- Basics
- Handling analysis

Basics

VEHICLE DYNAMICS STUDY

GOALS:

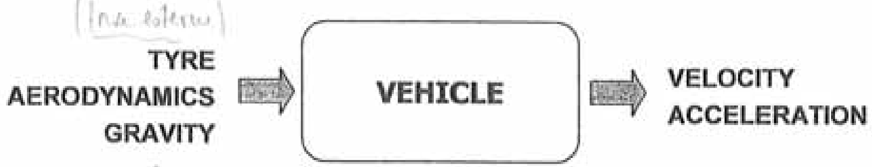
- Vehicles behaviour analysis and improvement in terms of:
  - Vehicle directional control stability
  - Roadholding
  - Steerability
  - Maneuverability
- ACTIVE SAFETY Increase
- Better users pleasure

VEHICLE DYNAMICS

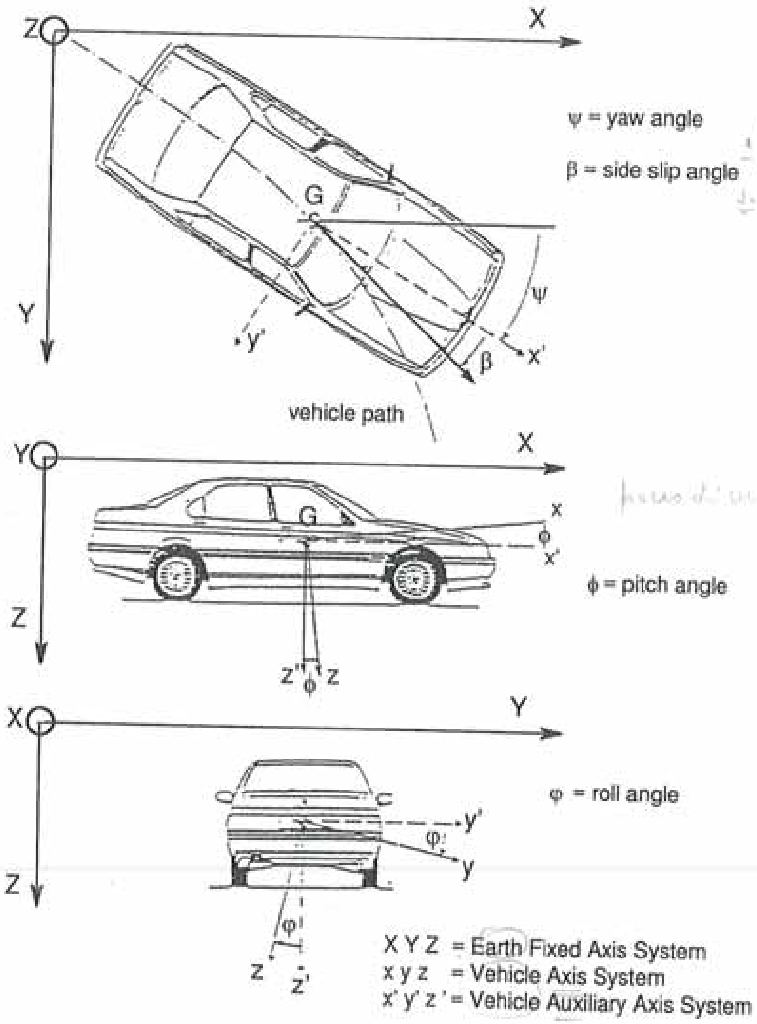
STUDIES VEHICLE MOTION WHEN EXTERNAL FORCES ARE APPLIED

FORCES APPLIED TO THE VEHICLE:

- Ground-Tyre Forces
- Aerodynamic forces on vehicle due to relative air motion
- Gravitational force component



VEHICLE AXIS SYSTEMS AND VEHICLE MOTIONS



VEHICLE AXIS SYSTEMS AND VEHICLE MOTIONS

- Sprung mass: All mass which is supported by the suspension, including portions of the mass of the suspension members
- Unsprung mass: All mass which is not carried by the suspension system, but is supported by tyre or wheel, and considered to move with it
- Three basic axis systems:
  - Earth-Fixed Axis System XYZ
  - Vehicle Axis System xyz (fixed in the vehicle body and moving with it)
  - Vehicle Auxiliary Axis System x'y'z'
- Lateral Acceleration Ay: is the component of the vector acceleration of the vehicle CG parallel to the y' direction
- Yaw Angle ψ : is the angle between the Earth-Fixed X-axis and the vehicle auxiliary x'-axis
- Sideslip Angle β: is the angle between the vehicle auxiliary x'-axis and the vehicle velocity vector
- Vehicle Pitch Angle ϕ: is the angle between the vehicle x-axis and the vehicle auxiliary x'-axis
- Vehicle Roll Angle θ: is the angle between the vehicle y-axis and the vehicle auxiliary y'-axis

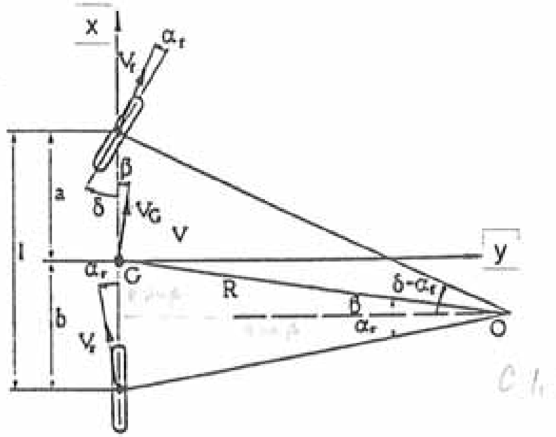
Handling analysis

HANDLING ANALYSIS : motion equations statement

STEADY STATE MANOEUVRES	TRANSIENT STATE MANOEUVRES
STEERING PAD	DRIVER INPUT TIME HISTORIES ARE IMPOSED
• CONSTANT PATH RADIUS • CONSTANT SPEED	Eg. STEP STEERING INPUT, TIP OUT ....
NON LINEAR ALGEBRIC EQUATIONS SYSTEM	NON LINEAR DIFFERENTIAL EQUATIONS SYSTEM

"BICYCLE MODEL"

- No lateral and longitudinal load transfer
- No rolling or pitching motion
- No chassis or suspension compliance effects
- No aerodynamic effects
- Position control (front wheel angle)
- Linear range tyres



STEADY STATE CORNERING

$$\frac{b - R\sin\beta}{R\cos\beta} = tg\,\alpha_r \quad \text{if} \quad \beta, \alpha_r \Rightarrow 0 \quad \text{then} \quad \frac{b}{R} - \beta \approx \alpha_r$$

$$\boxed{\beta \approx \frac{b}{R} - \alpha_r}$$

$$\frac{a + R\sin\beta}{R\cos\beta} = tg(\delta - \alpha_f) \quad \text{if} \quad \beta, \alpha_f \Rightarrow 0 \quad \text{then} \quad \frac{a}{R} + \frac{b}{R} - \alpha_r = \delta - \alpha_f$$

$$\boxed{\delta \approx \frac{L}{R} + \alpha_f - \alpha_r}$$

BASICS OF VEHICLE DYNAMICS
Module H1
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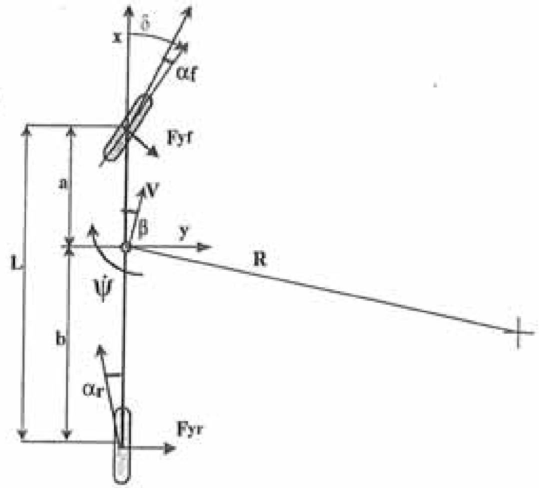
YAW EQUILIBRIUM

F_{yf} :lateral cornering force at front axle [N]
F_{yr} :lateral cornering force at rear axle [N]
$C\alpha f$: Cornering stiffness at front axle [N/rad]
$C\alpha r$: Cornering stiffness at rear axle [N/rad]
m : Mass of the vehicle [kg]
V : Forward velocity [m/s]
R : Radius of the turn [m]
a : longitudinal distance between front axle and center of gravity [m]
δ : Steer angle [rad]
δ_{vol} : Steering wheel angle [rad]
τ : Steering ratio



$(\delta \approx 0 \implies \cos\delta \approx 1)$

Lateral direction: $\sum F_y = F_{yf} + F_{yr} = Fc$

About the center of gravity: $\sum M_z = F_{yf} \cdot a - F_{yr} \cdot b = 0$

where

$$Fc = m \cdot a_y = m \cdot \dot\psi^2 \cdot R = m \cdot \frac{V^2}{R}$$

Solving the cornering equations yields:

$$F_{yf} = m \cdot \frac{V^2}{R} \cdot \frac{b}{L} = m_f \cdot a_y \qquad F_{yr} = m \cdot \frac{V^2}{R} \cdot \frac{a}{L} = m_r \cdot a_y$$

(YAW EQUILIBRIUM)

Supposing :

$$F_y = C \cdot \alpha$$

$$\alpha_f = \frac{F_{yf}}{C_f} = \frac{m_f}{C_f} \cdot \frac{V^2}{R} \qquad \alpha_r = \frac{F_{yr}}{C_r} = \frac{m_r}{C_r} \cdot \frac{V^2}{R}$$

$$\boxed{\delta_{vol} = \frac{L}{R} \cdot \tau - (\alpha_f - \alpha_r) \cdot \tau = \frac{L}{R} \cdot \tau + \left(\frac{m_f}{C_f} - \frac{m_r}{C_r}\right) \cdot \frac{V^2}{R} \cdot \tau = \delta_{vol0} + KUS \cdot a_y}$$

$$KUS = \left(\frac{m_f}{C_f} - \frac{m_r}{C_r}\right) \cdot \tau \quad \Rightarrow \text{Understeer gradient } [\text{rad}/\tfrac{\text{m}}{\text{s}^2}]$$

$$\boxed{\beta = \frac{b}{R} - \alpha_r = \frac{b}{R} - \frac{m_r}{C_r} \cdot \frac{V^2}{R} = \beta_0 - KBETA \cdot a_y}$$

$$KBETA = \frac{m_r}{C_r} \quad \Rightarrow \text{Sideslip angle gradient } [\text{rad}/\tfrac{\text{m}}{\text{s}^2}]$$

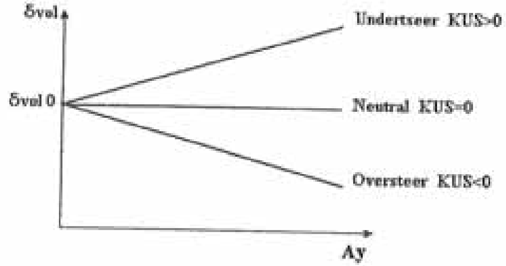
$$\beta = \frac{b}{R} - \frac{m_r}{C_r} \cdot \frac{V^2}{R} = \frac{b}{V^2} \cdot a_y - \frac{m_r}{C_r} \cdot a_y = \left(\frac{b}{V^2} - \frac{m_r}{C_r}\right) \cdot a_y$$

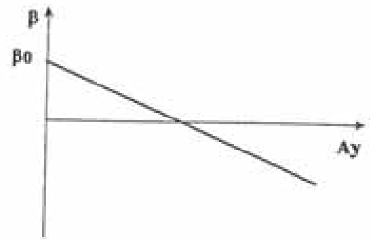
$$\left(\frac{b}{V^2} - \frac{m_r}{C_r}\right) = 0 \qquad V = \sqrt{\frac{C_r \cdot b}{m_r}}$$

Speed at which β=0

(YAW EQUILIBRIUM)

•Steering pad constant radius





Lateral acceleration gain :

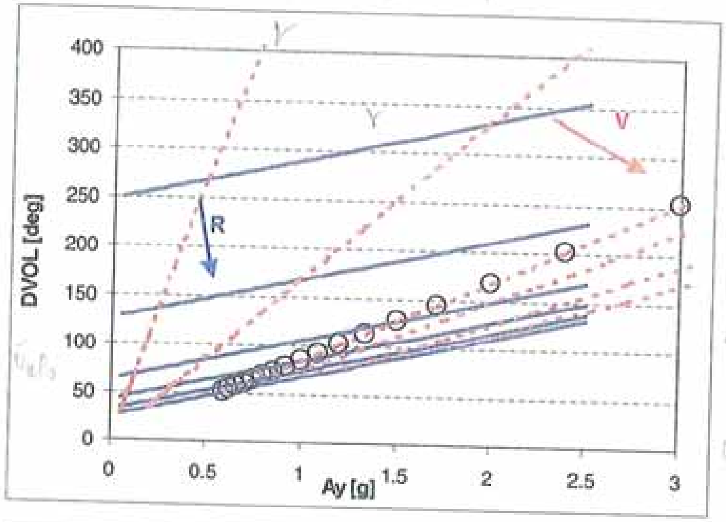
$$\frac{a_y}{\delta_{vol}} = \frac{1}{\frac{L}{V^2} \cdot \tau + KUS}$$

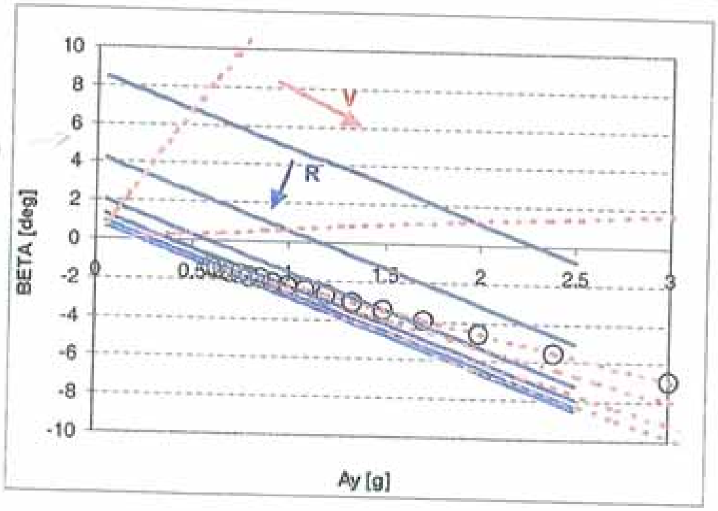
Yaw velocity gain :

$$\frac{\dot{\psi}}{\delta_{vol}} = \frac{1}{\frac{L}{V} \cdot \tau + KUS \cdot V}$$

(YAW EQUILIBRIUM:)

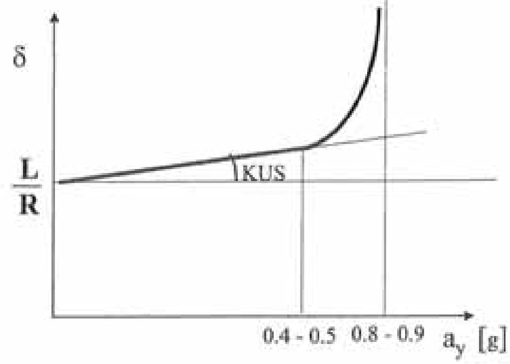
•*UNDERSTEER VEHICLE*



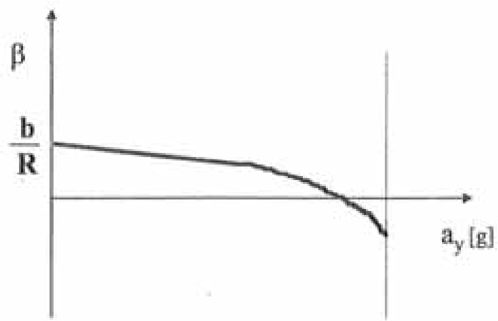


(YAW EQUILIBRIUM: NON LINEAR CASE)

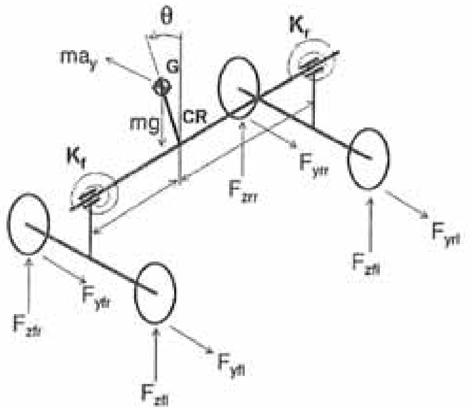
- Steering angle vs lateral acceleration

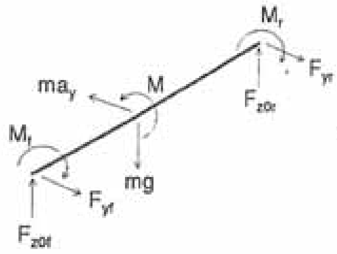


- Sideslip angle vs lateral acceleration



ROLL EQUILIBRIUM





h_{CR} = roll centre height

h_G = mass centre height

$\Delta h = h_G - h_{CR}$

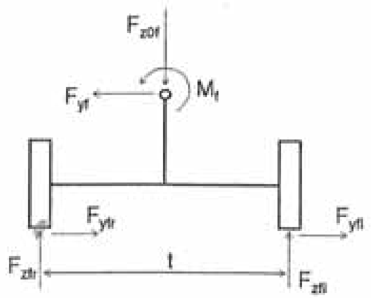
$$\begin{cases} F_{z0f} = m \cdot g \cdot \dfrac{b}{a+b} \\ F_{z0r} = m \cdot g \cdot \dfrac{a}{a+b} \end{cases}$$

$$\begin{cases} F_{yf} = m \cdot a_y \cdot \dfrac{b}{a+b} \\ F_{yr} = m \cdot a_y \cdot \dfrac{a}{a+b} \end{cases}$$

just suppose that: $\theta \equiv 0$

$$M = M_f + M_r = (K_f + K_r) \cdot \theta = m \cdot a_y \cdot \Delta h$$

$$\theta = \frac{m \cdot a_y \cdot \Delta h}{K_f + K_r}$$



$$F_{zfl} \cdot t + F_{yf} \cdot h_{CR} + M_f - F_{z0f} \cdot \frac{t}{2} = 0$$

$$F_{zfl} = \frac{1}{t} \cdot \left[F_{z0f} \cdot \frac{t}{2} - M_f - F_{yf} \cdot h_{CR} \right]$$

$$F_{zfl} = \frac{1}{t} \cdot \left[F_{z0f} \cdot \frac{t}{2} - K_f \cdot \theta - F_{yf} \cdot h_{CR} \right]$$

BASICS OF VEHICLE DYNAMICS
Module H1
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(ROLL EQUILIBRIUM)

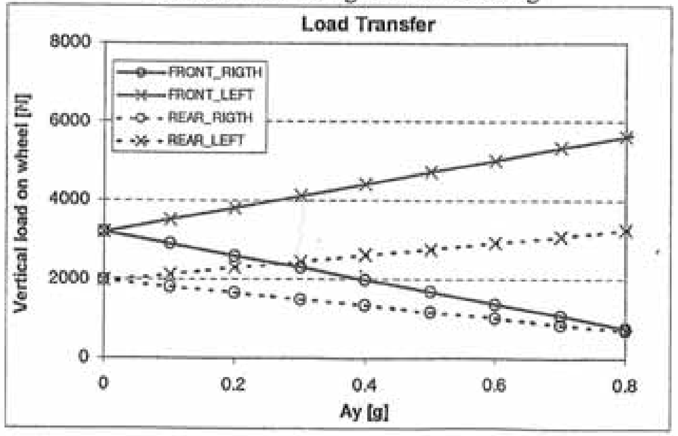
$$\begin{cases} F_{zfl} = \dfrac{m \cdot g}{2} \cdot \dfrac{b}{a+b} - m \cdot a_y \cdot \left(\dfrac{K_f}{K_f+K_r} \cdot \dfrac{\Delta h}{t} + \dfrac{b}{a+b} \cdot \dfrac{h_{CR}}{t} \right) \\ F_{zfr} = \dfrac{m \cdot g}{2} \cdot \dfrac{b}{a+b} + m \cdot a_y \cdot \left(\dfrac{K_f}{K_f+K_r} \cdot \dfrac{\Delta h}{t} + \dfrac{b}{a+b} \cdot \dfrac{h_{CR}}{t} \right) \\ F_{zrl} = \dfrac{m \cdot g}{2} \cdot \dfrac{a}{a+b} - m \cdot a_y \cdot \left(\dfrac{K_r}{K_f+K_r} \cdot \dfrac{\Delta h}{t} + \dfrac{a}{a+b} \cdot \dfrac{h_{CR}}{t} \right) \\ F_{zrr} = \dfrac{m \cdot g}{2} \cdot \dfrac{a}{a+b} + m \cdot a_y \cdot \left(\dfrac{K_r}{K_f+K_r} \cdot \dfrac{\Delta h}{t} + \dfrac{a}{a+b} \cdot \dfrac{h_{CR}}{t} \right) \end{cases}$$

Load transfer on single wheels

$$\begin{cases} F_{zfr} + F_{zrr} = \dfrac{m \cdot g}{2} + m \cdot a_y \cdot \dfrac{h_G}{t} \\ F_{zfl} + F_{zrl} = \dfrac{m \cdot g}{2} - m \cdot a_y \cdot \dfrac{h_G}{t} \end{cases}$$

Load transfer on the right and left side

•Constant radius right hand cornering



TRANSIENT BEHAVIOUR

Motion equation: $\overline{F} = \frac{d\overline{L}}{dt} \quad , \quad \overline{M} = \frac{d\overline{H}}{dt}$

$$\overline{L} = m \cdot \overline{V} \quad , \quad \overline{H} = J_z \cdot \psi$$

$$\left(\left.\frac{d\overline{A}}{dt}\right|_i = \left.\frac{d\overline{A}}{dt}\right|_m + \overline{\omega} \wedge \overline{A} \right)$$

$$\overline{F} = \frac{d\overline{L}}{dt} = \frac{d(m \cdot \overline{V})}{dt} + \overline{\dot{\psi}} \wedge \overline{L}$$

$$\overline{F} = \frac{d\overline{L}}{dt} = F_x \cdot \overline{i} + F_y \cdot \overline{j} = m \cdot \left(\frac{dV_x}{dt}\overline{i} + \frac{dV_y}{dt}\overline{j} \right) - \left(m \cdot V_y \cdot \dot{\psi} \cdot \overline{i} - m \cdot V_x \cdot \dot{\psi} \cdot \overline{j}\right)$$

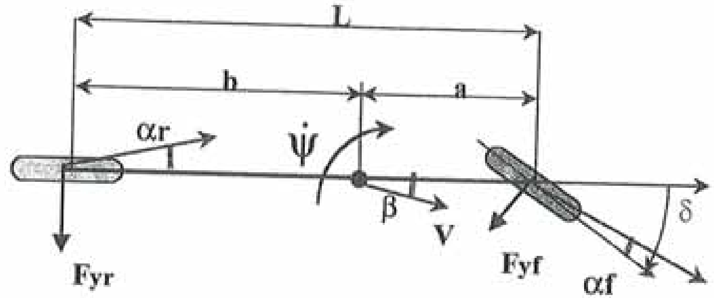
just suppose that: $V_x = V,\ V_y = V \cdot \beta$

$$\sum F_x = m \cdot \left(\dot{V} - V \cdot \dot{\psi} \cdot \beta\right) = m \cdot a_x$$
$$\sum F_y = m \cdot \left(\dot{V} \cdot \beta + V \cdot \dot{\beta} + V \cdot \dot{\psi}\right) = m \cdot a_y$$

$$\overline{M} = \frac{d\overline{H}}{dt} = \frac{d\left(J_z \cdot \overline{\dot{\psi}}\right)}{dt} + \overline{\dot{\psi}} \wedge \overline{H} = J_z \cdot \overline{\ddot{\psi}} \qquad \left(\overline{\dot{\psi}} \wedge \overline{H} = 0\right)$$

$$\sum M_z = J_z \cdot \ddot{\psi}$$

Lateral forces equilibrium and yaw equilibrum



$$F_{y'r} + F_{y'f} \cdot \cos\delta = m \cdot \left(\dot{V} \cdot \beta + V \cdot \dot{\beta} - V \cdot \dot{\psi}\right)$$
$$-b \cdot F_{y'r} + a \cdot F_{y'f} \cdot \cos\delta = J_z \cdot \ddot{\psi}$$

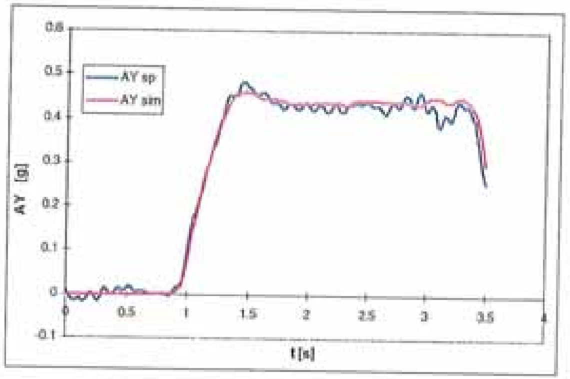
$$F_{y'f} = C_f \cdot \alpha_f \qquad F_{y'r} = C_r \cdot \alpha_r$$

$$\alpha_f = \delta - \beta - \frac{\dot{\psi} \cdot a}{V} \qquad \alpha_r = -\beta + \frac{\dot{\psi} \cdot b}{V}$$

V=Constant ⟹ $\dot{V} = 0$
δ ~0 ⟹ cosδ~1

$$C_r \cdot \alpha_r + C_f \cdot \alpha_f = m \cdot \left(V \cdot \dot{\beta} - V \cdot \dot{\psi}\right)$$
$$-b \cdot C_r \cdot \alpha_r + a \cdot C_f \cdot \alpha_f = J_z \cdot \ddot{\psi}$$

Example:


AY sp
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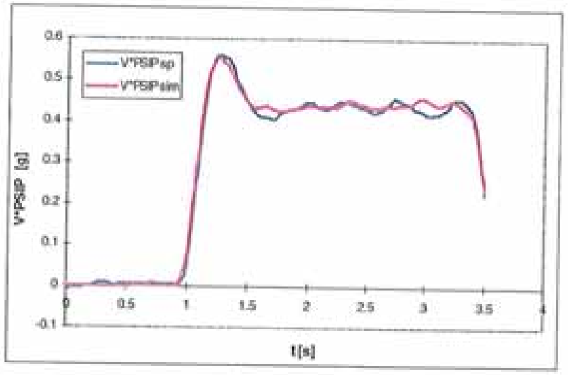
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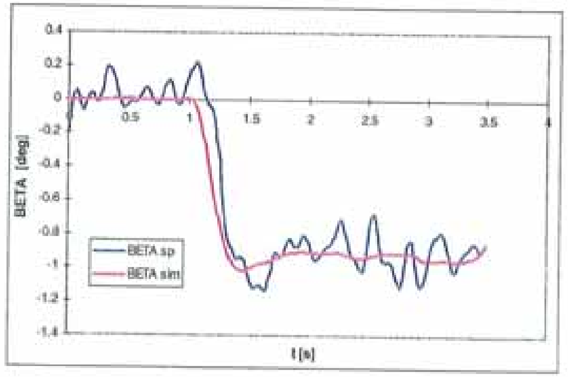
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